Dérivées 1

1°	$\left(x^n\right)' = n \cdot x^{n-1}$
2°	$(k)' = 0 \quad (k \in \mathbb{R})$
3°	$\left(f+g\right)' = f' + g'$
4°	$\left(k \cdot f\right)' = k \cdot f' \quad (k \in \mathbb{R})$

Ex 1 Calculer la dérivée

a) $\left(x^5\right)' =$

b) $\left(-5\right)' =$

c) $\left(x\right)' =$

d) $\left(x^4 + x^7\right)' =$

e) $\left(3x^6\right)' =$

f) $\left(-7x\right)' =$

g) $\left(2x^4 - 8x^3\right)' =$

h) $\left(6x^3 + 2x^2 + x + 1\right)' =$

i) $\left(7x^4 - 3x^2 - 4x + 9\right)' =$

j) $\left(3\left(x^2 + x\right)\right)' =$

k) $\left((x+3)(x-5)\right)' =$

l) $\left((x-4)^2\right)' =$

m) $\left((x-2)(x-3)(x+4)\right)' =$

Dérivées 1

1°	$\left(x^n\right)' = n \cdot x^{n-1}$
2°	$(k)' = 0 \quad (k \in \mathbb{R})$
3°	$\left(f+g\right)' = f' + g'$
4°	$\left(k \cdot f\right)' = k \cdot f' \quad (k \in \mathbb{R})$

Ex 1 Calculer la dérivée

a) $\left(x^5\right)' \underset{1°}{=} \underline{\underline{5x^4}}$

b) $\left(-5\right)' \underset{2°}{=} \underline{\underline{0}}$

c) $\left(x\right)' \underset{1°}{=} \underline{\underline{1}}$

d) $\left(x^4+x^7\right)' \underset{3°}{=} (x^4)' + (x^7)' \underset{1°}{=} \underline{\underline{4x^3+7x^6}}$

e) $\left(3x^6\right)' \underset{4°}{=} 3 \cdot (x^6)' \underset{1°}{=} 3 \cdot 6x^5 = \underline{\underline{18x^5}}$

f) $\left(-7x\right)' \underset{4°}{=} -7 \cdot (x)' \underset{1°}{=} -7 \cdot 1 = \underline{\underline{-7}}$

g) $\left(2x^4-8x^3\right)' \underset{3°}{=} (2x^4)' - (8x^3)' \underset{4°}{=} 2 \cdot (x^4)' - 8 \cdot (x^3)' \underset{1°}{=} 2 \cdot 4x^3 - 8 \cdot 3x^2 =$
$= \underline{\underline{8x^3 - 24x^2}}$

h) $\left(6x^3+2x^2+x+1\right)' \underset{3°/4°/1°/2°}{=} 18x^2 + 4x + 1 + 0 = \underline{\underline{18x^2+4x+1}}$

i) $\left(7x^4-3x^2-4x+9\right)' = \underline{\underline{28x^3 - 6x - 4}}$

j) $\left(3\left(x^2+x\right)\right)' = (3x^2+3x)' = \underline{\underline{6x+3}} = \underline{\underline{3(2x+1)}}$

k) $\left((x+3)(x-5)\right)' = (x^2-2x-15)' = \underline{\underline{2x-2}}$

l) $\left((x-4)^2\right)' = (x^2-8x+16)' = \underline{\underline{2x-8}}$

m) $\left(\underbrace{(x-2)(x-3)}_{(x^2-5x+6)}(x+4)\right)' = [x^3-5x^2+6x+4x^2-20x+24]' =$
$= [x^3-x^2-14x+24]' = \underline{\underline{3x^2-2x-14}}$

Dérivées 2

1°	$\left(x^n\right)' = n \cdot x^{n-1}$		5°	$\left(u \cdot v\right)' = u' \cdot v + u \cdot v'$
2°	$(k)' = 0 \quad (k \in \mathbb{R})$			
3°	$\left(u + v\right)' = u' + v'$			
4°	$\left(k \cdot u\right)' = k \cdot u' \quad (k \in \mathbb{R})$			

Ex 2 Calculer la dérivée

a) $f(x) = \underbrace{(x^2+3)}_{u(x)}\underbrace{(x^3-5x^2)}_{v(x)}$

$u(x) = x^2 + 3 \qquad v(x) = x^3 - 5x^2$

$u'(x) = \qquad v'(x) =$

$f'(x) \underset{5°}{=} u'(x) \cdot v(x) + u(x) \cdot v'(x)$

$f'(x) \underset{5°}{=}$

b) $f(x) = (3x^2 - x)(5x - 2)$

$u(x) = \qquad v(x) =$

$u'(x) = \qquad v'(x) =$

$f'(x) \underset{5°}{=}$

c) $f(x) = (x^3 - 8)(x^4 - 5x^2)$

$f'(x) \underset{5°}{=}$

d) $f(x) = (5x - 8)(6x^2 - x)$

Dérivées 2

1°	$\left(x^n\right)' = n \cdot x^{n-1}$		5°	$(u \cdot v)' = u' \cdot v + u \cdot v'$
2°	$(k)' = 0 \quad (k \in \mathbb{R})$			
3°	$(u+v)' = u' + v'$			
4°	$(k \cdot u)' = k \cdot u' \quad (k \in \mathbb{R})$			

Ex 2 Calculer la dérivée

a) $f(x) = \underbrace{(x^2+3)}_{u(x)}\underbrace{(x^3-5x^2)}_{v(x)}$

$u(x) = x^2 + 3 \qquad v(x) = x^3 - 5x^2$

$u'(x) = 2x \qquad v'(x) = 3x^2 - 10x$

$f'(x) \underset{5°}{=} u'(x) \cdot v(x) + u(x) \cdot v'(x)$

$f'(x) \underset{5°}{=} 2x \cdot (x^3 - 5x^2) + (x^2+3)(3x^2 - 10x) =$

$= 2x^4 - 10x^3 + 3x^4 - 10x^3 + 9x^2 - 30x =$

$= 5x^4 - 20x^3 + 9x^2 - 30x$

b) $f(x) = \underbrace{(3x^2 - x)}_{u}\underbrace{(5x-2)}_{v}$

$u(x) = 3x^2 - x \qquad v(x) = 5x - 2$

$u'(x) = 6x - 1 \qquad v'(x) = 5$

$f'(x) \underset{5°}{=} (6x-1)(5x-2) + (3x^2 - x) \cdot 5 =$

$= 30x^2 - 12x - 5x + 2 + 15x^2 - 5x =$

$= 45x^2 - 22x + 2$

c) $f(x) = \underbrace{(x^3-8)}_{u}\underbrace{(x^4-5x^2)}_{v}$

$u(x) = x^3 - 8 \qquad v(x) = x^4 - 5x^2$

$u'(x) = 3x^2 \qquad v'(x) = 4x^3 - 10x$

$f'(x) \underset{5°}{=} 3x^2(x^4 - 5x^2) + (x^3-8)(4x^3 - 10x) =$

$= 3x^6 - 15x^4 + 4x^6 - 10x^4 - 32x^3 + 80x =$

$= 7x^6 - 25x^4 - 32x^3 + 80x$

d) $f(x) = (5x-8)(6x^2 - x)$

$u(x) = 5x - 8 \qquad v(x) = 6x^2 - x$

$u'(x) = 5 \qquad v'(x) = 12x - 1$

$f'(x) = 5 \cdot (6x^2 - x) + (5x-8)(12x-1) =$

$= 30x^2 - 5x + 60x^2 - 5x - 96x + 8 =$

$= 90x^2 - 106x + 8$

Dérivées 3

1°	$\left(x^n\right)' = n \cdot x^{n-1}$	5°	$\left(u \cdot v\right)' = u' \cdot v + u \cdot v'$
2°	$(k)' = 0 \quad (k \in \mathbb{R})$	5°bis	$\left(u \cdot v \cdot w\right)' = u' \cdot v \cdot w + u \cdot v' \cdot w + u \cdot v \cdot w'$
3°	$\left(u + v\right)' = u' + v'$		
4°	$\left(k \cdot u\right)' = k \cdot u' \quad (k \in \mathbb{R})$		

Ex 3 Calculer la dérivée

a) $f(x) = \underbrace{(2x+3)}_{u(x)}\underbrace{(4x^2-5)}_{v(x)}\underbrace{(x-6)}_{w(x)}$

$u(x) = 2x+3 \qquad v(x) = 4x^2 - 5 \qquad w(x) = x - 6$

$u'(x) = \qquad v'(x) = \qquad w'(x) =$

$$f'(x) \underset{5°bis}{=} u'(x) \cdot v(x) \cdot w(x) + u(x) \cdot v'(x) \cdot w(x) + u(x) \cdot v(x) \cdot w'(x)$$

$$f'(x) \underset{5°}{=}$$

b) $f(x) = (x-4)(5x-2)(6-x)$

Dérivées 3

1°	$\left(x^n\right)' = n \cdot x^{n-1}$	5°	$\left(u \cdot v\right)' = u' \cdot v + u \cdot v'$
2°	$(k)' = 0 \quad (k \in \mathbb{R})$	5°bis	$\left(u \cdot v \cdot w\right)' = u' \cdot v \cdot w + u \cdot v' \cdot w + u \cdot v \cdot w'$
3°	$\left(u + v\right)' = u' + v'$		
4°	$\left(k \cdot u\right)' = k \cdot u' \quad (k \in \mathbb{R})$		

Ex 3 Calculer la dérivée

a) $f(x) = \underbrace{(2x+3)}_{u(x)}\underbrace{(4x^2-5)}_{v(x)}\underbrace{(x-6)}_{w(x)}$

$u(x) = 2x+3 \qquad v(x) = 4x^2-5 \qquad w(x) = x-6$

$u'(x) = 2 \qquad v'(x) = 8x \qquad w'(x) = 1$

$f'(x) \underset{5^\circ bis}{=} u'(x) \cdot v(x) \cdot w(x) + u(x) \cdot v'(x) \cdot w(x) + u(x) \cdot v(x) \cdot w'(x)$

$f'(x) \underset{5^\circ bis}{=} 2 \cdot \underbrace{(4x^2-5)(x-6)} + (2x+3) \cdot \underbrace{8x \cdot (x-6)} + (2x+3)(4x^2-5) \cdot 1 =$

$= 2\left[4x^3 - 24x^2 - 5x + 30\right] + (2x+3)\left[8x^2 - 48x\right] + (8x^3 - 10x + 12x^2 - 15) =$

$= 8x^3 - 48x^2 - 10x + 60 + 16x^3 - 96x^2 + 24x^2 - 144x + 8x^3 - 10x + 12x^2 - 15$

$= 32x^3 - 108x^2 - 164x + 45$

b) $f(x) = \underbrace{(x-4)}_{u}\underbrace{(5x-2)}_{v}\underbrace{(6-x)}_{w}$

$u(x) = x-4 \qquad v(x) = 5x-2 \qquad w(x) = 6-x$

$u'(x) = 1 \qquad v'(x) = 5 \qquad w'(x) = -1$

$f'(x) = 1 \cdot (5x-2)(6-x) + \underbrace{(x-4) \cdot 5} \cdot (6-x) + \underbrace{(x-4)(5x-2)} \cdot (-1) =$

$= 30x - 12 - 5x^2 + 2x + (5x-20)(6-x) + (5x^2 - 2x - 20x + 8)(-1) =$

$= -5x^2 + 32x - 12 + 30x - 5x^2 - 120 + 20x - 5x^2 + 22x - 8 =$

$= -15x^2 + 104x - 140$

Dérivées 4

1°	$\left(x^n\right)' = n \cdot x^{n-1}$		5°	$(u \cdot v)' = u' \cdot v + u \cdot v'$
2°	$(k)' = 0 \quad (k \in \mathbb{R})$		5°bis	$(u \cdot v \cdot w)' = u' \cdot v \cdot w + u \cdot v' \cdot w + u \cdot v \cdot w'$
3°	$(u+v)' = u' + v'$		6°	$\left(\dfrac{u}{v}\right)' = \dfrac{u' \cdot v - u \cdot v'}{v^2}$
4°	$(k \cdot u)' = k \cdot u' \quad (k \in \mathbb{R})$			

Ex 4 Calculer la dérivée

a) $f(x) = \dfrac{\overbrace{x^2 - 5x}^{u(x)}}{\underbrace{3x-2}_{v(x)}}$

$u(x) = x^2 - 5x$ $\qquad$ $v(x) = 3x - 2$

$u'(x) =$ $\qquad$ $v'(x) =$

$$f'(x) \underset{6°}{=} \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v^2(x)}$$

$f'(x) \underset{6°}{=}$

b) $f(x) = \dfrac{3x^2 - 5}{x^2 + x}$

$u(x) =$ $\qquad$ $v(x) =$

$u'(x) =$ $\qquad$ $v'(x) =$

$f'(x) \underset{6°}{=}$

Dérivées 4

1°	$\left(x^n\right)' = n \cdot x^{n-1}$		5°	$(u \cdot v)' = u' \cdot v + u \cdot v'$
2°	$(k)' = 0 \quad (k \in \mathbb{R})$		5°bis	$(u \cdot v \cdot w)' = u' \cdot v \cdot w + u \cdot v' \cdot w + u \cdot v \cdot w'$
3°	$(u+v)' = u' + v'$		6°	$\left(\dfrac{u}{v}\right)' = \dfrac{u' \cdot v - u \cdot v'}{v^2}$
4°	$(k \cdot u)' = k \cdot u' \quad (k \in \mathbb{R})$			

Ex 4 Calculer la dérivée

a) $f(x) = \dfrac{\overbrace{x^2 - 5x}^{u(x)}}{\underbrace{3x-2}_{v(x)}}$ $\qquad u(x) = x^2 - 5x \qquad v(x) = 3x - 2$

$u'(x) = 2x - 5 \qquad v'(x) = 3$

$$f'(x) \underset{6°}{=} \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{v^2(x)}$$

$$f'(x) \underset{6°}{=} \frac{(2x-5)(3x-2) - (x^2-5x) \cdot 3}{(3x-2)^2} =$$

$$= \frac{6x^2 - 4x - 15x + 10 - 3x^2 + 15x}{(3x-2)^2} =$$

$$= \frac{3x^2 - 4x + 10}{(3x-2)^2}$$

b) $f(x) = \dfrac{3x^2 - 5}{x^2 + x}$ $\qquad u(x) = 3x^2 - 5 \qquad v(x) = x^2 + x$

$u'(x) = 6x \qquad v'(x) = 2x + 1$

$$f'(x) \underset{6°}{=} \frac{6x \cdot (x^2+x) - (3x^2-5)(2x+1)}{(x^2+x)^2} =$$

$$= \frac{6x^3 + 6x^2 - [6x^3 + 3x^2 - 10x - 5]}{(x^2+x)^2} =$$

$$= \frac{6x^3 + 6x^2 - 6x^3 - 3x^2 + 10x + 5}{(x^2+x)^2} =$$

$$= \frac{3x^2 + 10x + 5}{(x^2+x)^2} \underset{*}{=} \frac{3x^2 + 10x + 5}{x^2(x+1)^2}$$

$*$ $x^2 + x = x(x+1)$

$\Rightarrow (x^2+x)^2 = x^2(x+1)^2$

Dérivées 5

1°	$\left(x^n\right)' = n \cdot x^{n-1}$	5°	$\left(u \cdot v\right)' = u' \cdot v + u \cdot v'$
2°	$(k)' = 0 \quad (k \in \mathbb{R})$	5°bis	$\left(u \cdot v \cdot w\right)' = u' \cdot v \cdot w + u \cdot v' \cdot w + u \cdot v \cdot w'$
3°	$\left(u + v\right)' = u' + v'$	6°	$\left(\dfrac{u}{v}\right)' = \dfrac{u' \cdot v - u \cdot v'}{v^2}$
4°	$\left(k \cdot u\right)' = k \cdot u' \quad (k \in \mathbb{R})$	6°bis	$\left(\dfrac{1}{v}\right)' = \dfrac{-v'}{v^2}$

Ex 5 Calculer la dérivée

a) $f(x) = \dfrac{1}{\underbrace{3x^2 - 2x}_{v(x)}}$

$v(x) = 3x^2 - 2x$

$v'(x) =$

$f'(x) \underset{6°bis}{=} \dfrac{-v'(x)}{v^2(x)}$

$f'(x) \underset{6°bis}{=}$

b) $f(x) = \dfrac{1}{x}$

$v(x) =$

$v'(x) =$

$f'(x) \underset{6°bis}{=}$

c) $f(x) = \dfrac{1}{x-3} - \dfrac{1}{x+1}$

Dérivées 5

1°	$(x^n)' = n \cdot x^{n-1}$	5°	$(u \cdot v)' = u' \cdot v + u \cdot v'$
2°	$(k)' = 0 \quad (k \in \mathbb{R})$	5°bis	$(u \cdot v \cdot w)' = u' \cdot v \cdot w + u \cdot v' \cdot w + u \cdot v \cdot w'$
3°	$(u+v)' = u' + v'$	6°	$\left(\frac{u}{v}\right)' = \frac{u' \cdot v - u \cdot v'}{v^2}$
4°	$(k \cdot u)' = k \cdot u' \quad (k \in \mathbb{R})$	6°bis	$\left(\frac{1}{v}\right)' = \frac{-v'}{v^2}$

Ex 5 Calculer la dérivée

a) $f(x) = \frac{1}{\underbrace{3x^2 - 2x}_{v(x)}}$

$v(x) = 3x^2 - 2x$

$v'(x) = 6x - 2$

$f'(x) \underset{6°bis}{=} \frac{-v'(x)}{v^2(x)}$

$f'(x) \underset{6°bis}{=} \frac{-(6x-2)}{(3x^2-2x)^2} \underset{*}{=} \frac{-2(3x-1)}{x^2(3x-2)^2}$

$* \quad 6x - 2 = 2(3x-1)$

$3x^2 - 2x = x(3x-2)$

$\Rightarrow (3x^2-2x)^2 = x^2(3x-2)^2$

b) $f(x) = \frac{1}{x}$

$v(x) = x$

$v'(x) = 1$

$f'(x) \underset{6°bis}{=} \frac{-1}{x^2} = -\frac{1}{x^2}$

c) $f(x) = \frac{1}{x-3} - \frac{1}{x+1}$

$g(x) = \frac{1}{x-3}$	$v(x) = x - 3$ $v'(x) = 1$
$g'(x) = \frac{-1}{(x-3)^2}$	
$h(x) = \frac{1}{x+1}$	$v(x) = x+1$ $v'(x) = 1$
$h'(x) = \frac{-1}{(x+1)^2}$	

$f'(x) = \frac{-1}{(x-3)^2} - \frac{-1}{(x+1)^2} =$

3°+6°bis

$= \frac{-1}{(x-3)^2} + \frac{1}{(x+1)^2} =$

$= \frac{-1 \cdot (x+1)^2}{(x-3)^2(x+1)^2} + \frac{1 \cdot (x-3)^2}{(x-3)^2(x+1)^2} = \frac{-(x^2+2x+1) + (x^2-6x+9)}{(x-3)^2(x+1)^2} =$

$= \frac{-x^2 - 2x - 1 + x^2 - 6x + 9}{(x-3)^2(x+1)^2} = \frac{-8x+8}{(x-3)^2(x+1)^2} = \frac{-8(x-1)}{(x-3)^2(x+1)^2}$

Dérivées 6

1°	$\left(x^n\right)' = n \cdot x^{n-1}$
2°	$(k)' = 0 \quad (k \in \mathbb{R})$
3°	$\left(u+v\right)' = u' + v'$
4°	$\left(k \cdot u\right)' = k \cdot u' \quad (k \in \mathbb{R})$

5°	$\left(u \cdot v\right)' = u' \cdot v + u \cdot v'$
5°bis	$\left(u \cdot v \cdot w\right)' = u' \cdot v \cdot w + u \cdot v' \cdot w + u \cdot v \cdot w'$
6°	$\left(\dfrac{u}{v}\right)' = \dfrac{u' \cdot v - u \cdot v'}{v^2}$
6°bis	$\left(\dfrac{1}{v}\right)' = \dfrac{-v'}{v^2}$
7°	$\left(u^n\right)' = n \cdot u^{n-1} \cdot u'$

Ex 6 Calculer la dérivée

a) $f(x) = \underbrace{(3x-2)}_{u(x)}^5$

$u(x) = 3x - 2$

$u'(x) =$

$f'(x) \underset{7°}{=} n \cdot (u(x))^{n-1} \cdot u'(x)$

$f'(x) \underset{7°}{=}$

b) $f(x) = (4x^2 - 5x + 3)^3$

$u(x) =$

$u'(x) =$

$f'(x) \underset{7°}{=}$

c) $f(x) = (5 - 4x)^2$

d) $f(x) = (5x - 8)^3 + (2x - 9)^2$

Dérivées 6

1°	$\left(x^n\right)' = n \cdot x^{n-1}$
2°	$(k)' = 0 \quad (k \in \mathbb{R})$
3°	$\left(u+v\right)' = u' + v'$
4°	$\left(k \cdot u\right)' = k \cdot u' \quad (k \in \mathbb{R})$

5°	$\left(u \cdot v\right)' = u' \cdot v + u \cdot v'$
5°bis	$\left(u \cdot v \cdot w\right)' = u' \cdot v \cdot w + u \cdot v' \cdot w + u \cdot v \cdot w'$
6°	$\left(\dfrac{u}{v}\right)' = \dfrac{u' \cdot v - u \cdot v'}{v^2}$
6°bis	$\left(\dfrac{1}{v}\right)' = \dfrac{-v'}{v^2}$
7°	$\left(u^n\right)' = n \cdot u^{n-1} \cdot u'$

Ex 6 Calculer la dérivée

a) $f(x) = \underbrace{(3x-2)}_{u(x)}{}^5$

$u(x) = 3x - 2$

$u'(x) = 3$

$f'(x) \underset{7°}{=} n \cdot (u(x))^{n-1} \cdot u'(x)$

$f'(x) \underset{7°}{=} 5 \cdot (3x-2)^4 \cdot 3 = \underline{\underline{15(3x-2)^4}}$

b) $f(x) = (4x^2 - 5x + 3)^3$

$u(x) = 4x^2 - 5x + 3$

$u'(x) = 8x - 5$

$f'(x) \underset{7°}{=} \underline{\underline{3 \cdot (4x^2 - 5x + 3)^2 \cdot (8x - 5)}}$

c) $f(x) = (5 - 4x)^2$

$u(x) = 5 - 4x$

$u'(x) = -4$

$f'(x) \underset{7°}{=} 2(5-4x) \cdot (-4) =$

$= \underline{\underline{-8(5-4x)}} = \underline{\underline{8(4x-5)}}$

d) $f(x) = (5x-8)^3 + (2x-9)^2$

$u(x) = 5x - 8$

$u'(x) = 5$

$u(x) = 2x - 9$

$u'(x) = 2$

$f'(x) = 3(5x-8)^2 \cdot 5 + 2(2x-9) \cdot 2 =$

$= 15(5x-8)^2 + 4(2x-9) =$

$= 15(25x^2 - 80x + 64) + 8x - 36 =$

$= 375x^2 - 1200x + 960 + 8x - 36 = \underline{\underline{375x^2 - 1192x + 924}}$

Dérivées 7

Révision

Ex 7 Calculer la dérivée

a) $f(x) = 3x^2 - 5x + 4$

b) $f(x) = (5x^2 - 2)(4x - 3)$

c) $f(x) = \dfrac{3x - 5}{x^2 - 2x}$

d) $f(x) = x(x-1)(x+3)$

e) $f(x) = \dfrac{1}{x^2 - 4}$

f) $f(x) = (4x^2 - 9)^3$

g) $f(x) = (5 - x)^2(2x - 7)$

h) $f(x) = \dfrac{(x-4)(x+2)}{x+3}$

i) $f(x) = \dfrac{(2x-3)^3}{(1-x)^2}$

j) $f(x) = 3{,}14 \cdot (x^2 - 2x)$

k) $f(x) = (\dfrac{\pi}{4} - 2) \cdot x^2 + 100 \cdot x$

l) $f(x) = 375x + \dfrac{150'000}{x}$

m) $f(x) = 6\left(x^2 + \dfrac{16}{x}\right)$

n) $f(x) = \dfrac{3x^3 + 139'200x + 6'000'000}{250x}$

Dérivées 7

Révision

Ex 7

a) $f(x) = 3x^2 - 5x + 4$

$\underline{f'(x) = 6x - 5}$

---

b) $f(x) = \underbrace{(5x^2 - 2)}_{u(x)}\underbrace{(4x-3)}_{v(x)}$

$u(x) = 5x^2 - 2 \qquad v(x) = 4x - 3$

$u'(x) = 10x \qquad v'(x) = 4$

$$\boxed{(u \cdot v)' = u' \cdot v + u \cdot v'}$$

$f'(x) = 10x \cdot (4x-3) + 4 \cdot (5x^2 - 2)$

$= 40x^2 - 30x + 20x^2 - 8$

$= \underline{\underline{60x^2 - 30x - 8}}$

---

c) $f(x) = \dfrac{3x-5}{x^2-2x}$

$u(x) = 3x - 5 \qquad v(x) = x^2 - 2x$

$u'(x) = 3 \qquad v'(x) = 2x - 2$

$$\boxed{\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}}$$

$f'(x) = \dfrac{3 \cdot (x^2 - 2x) - \overbrace{(3x-5)(2x-2)}^{6x^2 - 10x - 6x + 10}}{(x^2-2x)^2} =$

$= \dfrac{3x^2 - 6x - 6x^2 + 16x - 10}{(x^2-2x)^2} =$

$= \dfrac{-3x^2 + 10x - 10}{(x^2-2x)^2} = \underline{\underline{-\dfrac{3x^2 - 10x + 10}{x^2(x-2)^2}}}$

$(x^2-2x)^2 = (x(x-2))^2 = x^2(x-2)^2$

---

d) $f(x) = \underbrace{x}_{u(x)}\underbrace{(x-1)}_{v(x)}\underbrace{(x+3)}_{w(x)}$

$u(x) = x \qquad v(x) = x - 1 \qquad w(x) = x + 3$

$u'(x) = 1 \qquad v'(x) = 1 \qquad w'(x) = 1$

$$\boxed{(u \cdot v \cdot w)' = u' \cdot v \cdot w + u \cdot v' \cdot w + u \cdot v \cdot w'}$$

$f'(x) = 1 \cdot (x-1)(x+3) + x \cdot 1 \cdot (x+3) + x(x-1) \cdot 1 =$

$= x^2 - x + 3x - 3 + x^2 + 3x + x^2 - x =$

$= \underline{\underline{3x^2 + 4x - 3}}$

e) $f(x) = \frac{1}{x^2-4}$

$v(x) = x^2 - 4$

$v'(x) = 2x$

$$\boxed{\left(\frac{1}{v}\right)' = \frac{-v'}{v^2}}$$

$$f'(x) = \frac{-2x}{(x^2-4)^2}$$

---

f) $f(x) = (4x^2-9)^3$

$u(x) = 4x^2 - 9$ $\quad n = 3$

$u'(x) = 8x$

$$\boxed{(u^n)' = n \cdot u^{n-1} \cdot u'}$$

$$f'(x) = 3 \cdot (4x^2-9)^2 \cdot 8x = 24x\,(4x^2-9)^2$$

---

g) $f(x) = \underbrace{(5-x)^2}_{=u(x)} \underbrace{(2x-7)}_{=v(x)}$

$u(x) = (5-x)^2$ $\quad v(x) = 2x - 7$

$u'(x) \stackrel{1°}{=} 2 \cdot (5-x) \cdot (-1)$ $\quad v'(x) = 2$

⚠ dérivée interne

$$\boxed{(u \cdot v)' = u' \cdot v + u \cdot v'}$$

$$f'(x) = -2\underline{(5-x)}(2x-7) + \underline{(5-x)}^2 \cdot 2 =$$

$$= (5-x)\left[-2(2x-7) + 2(5-x)\right] =$$

⚠ mise en évidence de $(5-x)$

$$= (5-x)\left[-4x + 14 + 10 - 2x\right] =$$

$$= (5-x)(-6x+24) =$$

$$= -(x-5)(-6)(x-4) =$$

$$= 6(x-5)(x-4)$$

Ex 7 suite

h) $f(x) = \dfrac{(x-4)(x+2)}{x+3} \underset{\uparrow}{=} \dfrac{x^2-2x-8}{x+3}$

astuce !

$u(x) = x^2-2x-8 \qquad v(x) = x+3$

$u'(x) = 2x-2 \qquad v'(x) = 1$

$$\boxed{\left(\frac{u}{v}\right)' = \frac{u'v - u \cdot v'}{v^2}}$$

$$f'(x) = \frac{(2x-2)(x+3) - (x^2-2x-8)\cdot 1}{(x+3)^2} =$$

$$= \frac{2x^2+6x-2x-6-x^2+2x+8}{(x+3)^2} = \underline{\underline{\frac{x^2+6x+2}{(x+3)^2}}}$$

---

i) $f(x) = \dfrac{(2x-3)^3}{(1-x)^2}$

$u(x) = (2x-3)^3$

$u'(x) \underset{7°}{=} 3\cdot(2x-3)^2 \cdot 2 = 6(2x-3)^2$

$v(x) = (1-x)^2$

$v'(x) \underset{7°}{=} 2(1-x)(-1) = -2(1-x)$

$$f'(x) \underset{6°}{=} \frac{6(2x-3)^2(1-x)^2 - (2x-3)^3\cdot(-2)(1-x)}{(1-x)^4} =$$

$$\underset{\uparrow}{=} \frac{2(2x-3)^2(1-x)[3(1-x)+(2x-3)]}{(1-x)^4} \underset{\uparrow}{=}$$

⚠ mise en évidence de $2\cdot(2x-3)^2\cdot(1-x)$

simplification par $(1-x)$

$$= \frac{2(2x-3)^2[3-3x+2x-3]}{(1-x)^3} = \frac{2(2x-3)^2(-x)}{(1-x)^3} =$$

$$= \frac{-2x(2x-3)^2}{(1-x)^3} = \frac{-2x(2x-3)^2}{-(x-1)^3} = \underline{\underline{\frac{2x(2x-3)^2}{(x-1)^3}}}$$

j) $f(x) = 3{,}14 \cdot (x^2 - 2x)$

$$f'(x) \underset{4^\circ}{=} 3{,}14 \cdot [x^2 - 2x]' = 3{,}14 \cdot (\underset{\substack{\shortparallel \\ 2(x-1)}}{2x - 2}) =$$

$$\boxed{(k \cdot f)' = k \cdot f'}$$

$$= \underline{\underline{6{,}28\ (x-1)}}$$

---

k) $f(x) = \left(\frac{\pi}{4} - 2\right) x^2 + 100\,x$

$$f'(x) \underset{3^\circ}{=} \left[\left(\frac{\pi}{4} - 2\right) x^2\right]' + [100\,x]' =$$

$$\underset{4^\circ}{=} \left(\frac{\pi}{4} - 2\right) \cdot [x^2]' + 100 \cdot [x]' =$$

$$\underset{1^\circ}{=} \left(\frac{\pi}{4} - 2\right) \cdot 2x + 100 \cdot 1 =$$

$$= \underline{\underline{\left(\frac{\pi}{2} - 4\right) x + 100}}$$

---

l) $f(x) = 375\,x + \frac{150'000}{x}$

$$f'(x) \underset{3^\circ}{=} [375\,x]' + \left[\frac{150'000}{x}\right]' =$$

$$\underset{4^\circ}{=} 375 \cdot [x]' + 150'000 \cdot \left[\frac{1}{x}\right]' =$$

$\left(\frac{1}{x}\right)' = \frac{-1}{x^2}$
$\left(\frac{1}{v}\right)' = \frac{-v'}{v^2}$

$$\underset{1^\circ}{=} 375 \cdot 1 + 150'000 \cdot \frac{-1}{x^2} =$$

$$= 375 - \frac{150'000}{x^2} =$$

$$= \frac{375\,x^2 - 150'000}{x^2} = \frac{375\,(x^2 - 400)}{x^2} =$$

$$= \frac{375\,(x-20)(x+20)}{x^2}$$

Ex 7 suite et fin

m) $f(x) = 6\left(x^2 + \frac{16}{x}\right) = 6\,\frac{x^3+16}{x}$

$f'(x) \underset{4°}{=} 6 \cdot \left[\frac{x^3+16}{x}\right]' =$ $\quad u(x) = x^3+16 \quad v(x) = x$

$\quad u'(x) = 3x^2 \quad v'(x) = 1$

$$\underset{6°}{=} 6 \cdot \frac{3x^2 \cdot x - (x^3+16)\cdot 1}{x^2} =$$

$$= 6 \cdot \frac{3x^3 - x^3 - 16}{x^2} =$$

$$= 6 \cdot \frac{2x^3-16}{x^2} = 6 \cdot \frac{2(x^3-8)}{x^2} = \frac{12\,(x^3-8)}{x^2}$$

---

n) $f(x) = \frac{3x^3 + 139'200x + 6'000'000}{250x}$

$u(x) = 3x^3 + 139'200x + 6'000'000$

$u'(x) = 9x^2 + 139'200$

$v(x) = 250x$

$v'(x) = 250$

$$f'(x) \underset{6°}{=} \frac{(9x^2+139'200)\cdot 250x - (3x^3+139'200x+6'000'000)\cdot 250}{\underset{\underset{250 \cdot 250x^2}{\parallel}}{\underset{250x \cdot 250x}{\parallel}}{(250x)^2}} :$$

$$= \frac{\cancel{250}\left[(9x^2+139'200)\cdot x - (3x^3+139'200x+6'000'000)\right]}{\cancel{250}\cdot 250x^2} =$$

$$= \frac{9x^3 + 139'200x - 3x^3 - 139'200x - 6'000'000}{250x^2} = \frac{6x^3 - 6'000'000}{250x^2} :$$

$$= \frac{\overset{3}{\cancel{6}}(x^3 - 1'000'000)}{\cancel{250}x^2} = \frac{3\,(x^3-1'000'000)}{125\,x^2}$$

Dérivées

Préparation au test

a) $f(x) = 5x^3 - 7x^2 + 4x - 8$

b) $f(x) = 3{,}14x^2 - 6{,}28x + 1$

c) $f(x) = \pi x^2 - 2\pi x + 1$

d) $f(x) = (2x^2 - 5)(4 - x)$

e) $f(x) = 3'000(4x - 1)(5x - 8)$

f) $f(x) = (3x^3 + 5)^4$

g) $f(x) = 6(3x - 4)^2$

h) $f(x) = \dfrac{2x - 5}{x - 1}$

i) $f(x) = \dfrac{1}{x^2}$

j) $f(x) = 5x + \dfrac{125}{x}$

k) $f(x) = (x + 2)^3(x - 3)^2$

l) $f(x) = \dfrac{(x - 1)^2}{x + 2}$

m) $f(x) = \dfrac{3x^2}{2} - 40x + 400$

n) $f(x) = (48 - 2x)(30 - 2x)x$

o) $f(x) = (x + 6)(\dfrac{60}{x} + 10)$

Réponses :

a) $f'(x) = 15x^2 - 14x + 4$

b) $f'(x) = 6{,}28x - 6{,}28 = 6{,}28(x - 1)$

c) $f'(x) = 2\pi x - 2\pi = 2\pi(x - 1)$

d) $f'(x) = -6x^2 + 16x + 5$

e) $f'(x) = 3'000(40x - 37)$

f) $f'(x) = 36x^2(3x^3 + 5)^3$

g) $f'(x) = 36(3x - 4)$

h) $f'(x) = \dfrac{3}{(x - 1)^2}$

i) $f'(x) = \dfrac{-2}{x^3} = -\dfrac{2}{x^3}$

j) $f'(x) = 5 - \dfrac{125}{x^2} = \dfrac{5(x - 5)(x + 5)}{x^2}$

k) $f'(x) = 5(x - 3)(x - 1)(x + 2)^2$

l) $f'(x) = \dfrac{(x - 1)(x + 5)}{(x + 2)^2}$

m) $f'(x) = 3x - 40$

n) $f'(x) = 12x^2 - 312x + 1440 =$
$12(x - 20)(x - 6)$

o) $f'(x) = \dfrac{10(x - 6)(x + 6)}{x^2}$